(2)

A Final Technicial Report
Grant No. AFOSR-87-0321A

September 1, 1987 - November 30, 1989

INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

Submitted to:

Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, DC 20332

Attention:

Program Manager
Control Theory Program

Submitted by:

I. Lasiecka
Professor

R. Triggiani
Professor

Report No. UVA/525701/AM90/101
February 1990

DEPARTMENT OF APPLIED MATHEMATICS

SCHOOL OF
ENGINEERING
& APPLIED SCIENCE

University of Virginia
Thornton Hall
Charlottesville, VA 22903

UNIVERSITY OF VIRGINIA
School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,500 students with a graduate enrollment of approximately 600. There are 160 faculty members, a majority of whom conduct research in addition to teaching.

Research is a vital part of the educational program and interests parallel academic specialties. These range from the classical engineering disciplines of Chemical, Civil, Electrical, and Mechanical and Aerospace to newer, more specialized fields of Applied Mechanics, Biomedical Engineering, Systems Engineering, Materials Science, Nuclear Engineering and Engineering Physics, Applied Mathematics and Computer Science. Within these disciplines there are well equipped laboratories for conducting highly specialized research. All departments offer the doctorate; Biomedical and Materials Science grant only graduate degrees. In addition, courses in the humanities are offered within the School.

The University of Virginia (which includes approximately 2,000 faculty and a total of full-time student enrollment of about 17,000), also offers professional degrees under the schools of Architecture, Law, Medicine, Nursing, Commerce, Business Administration, and Education. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. The School of Engineering and Applied Science is an integral part of this University community which provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.

A Final Technical Report
Grant No. AFOSR-87-0321A

September 1, 198~~8~~7 - November 30, 1989

INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

Submitted to:

Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, DC 20332

Attention:

Program Manager
Control Theory Program

Submitted by:

I. Lasiecka
Professor

R. Triggiani
Professor

Department of Applied Mathematics
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VA 22903-2442

Report No. UVA/525701/AM90/101
February 1990

Copy No. 3

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave blank*)

2. REPORT DATE
2-22-90

3. REPORT TYPE AND DATES COVERED
Final Sept. 1, 1987 - Nov. 30, 1989

4. TITLE AND SUBTITLE
Increasing the Margin of Stability of Arbitrarily Finite Modes of Flexible Large Space Structures with Damping

5. FUNDING NUMBERS
AFOSR-87-0321A

6. AUTHOR(S)
R. Triggiani, Professor
I. Lasiecka, Professor

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Dept. of Applied Mathematics, School of Engineering and Applied Science, University of Virginia, Thornton Hall, Charlottesville, VA 22903-2442

8. PERFORMING ORGANIZATION REPORT NUMBER
UVA/525701/AM90/101

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, DC 20332

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unlimited Distribution

12b. DISTRIBUTION CODE

13.

This final report summarizes the principal investigators' achievements on the research project during the period September 1, 1987 through November 30, 1989. These include new results for wave equations and plate equations, linear and nonlinear, on the following problems: exact controllability, strong and uniform stabilization, structural damping, quadratic optimal control problem, Riccati equations, robustness with respect to nonlinear uncertainties, and numerical aspects thereof.

14. SUBJECT TERMS

15. NUMBER OF PAGES
13

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited

INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

During the period in question, the research activities of the principal investigators have continued along the lines summarized in Section I of the research proposal (continuation) submitted to the AFOSR in January 1987. Research efforts have centered on boundary control problems for waves and plates equations (in any dimension) with various boundary conditions, their abstract unifying representation introduced by the principal investigators, as well as related problems of structural damping for elastic systems. More precisely, major themes of research performed under the grant include the following problems:

(i) exact boundary controllability for waves and plates;

(ii) uniform stabilization (linear case) and strong stabilization (nonlinear case) by a-priori, explicit boundary feedback for waves and plates;

(iii) study of the optimal quadratic cost problem for waves and plates, in particular of the associated Algebraic Riccati Equation, which produces a boundary feedback based on the Riccati operator which uniformly stabilizes the system [compare with (ii)]; study of the robustness properties of such feedbacks.

(iv) Analyticity structural damping and differentiability/Gevrey class properties for elastic systems under a natural, broad class of damping operators; and

(v) numerical aspects related to some of the topics above.

Specific contributions in each of the above areas (i) - (v) will be separately reviewed below.

(i) Exact boundary controllability for waves and plates

Waves

It was already noted in Section I of the research proposal (continuation) submitted to the AFOSR in January 1987, that the first time that exact controllability of the wave equation with Dirichlet boundary control was obtained in its natural state space was in the principal investigators' paper [L-T.1], as a corollary of a related uniform stabilization result. Further contributions followed, by J. L. Lions and his associates, and by the principal investigators, see [T.2]. More recent contributions by the principal investigators include a rather thorough study of exact boundary controllability of wave equations with Neumann boundary control [L-T.3]: a study of these questions when the equation contains damping [T.5]. Finally, the P.I.'s have obtained new results on (global) exact controllability for abstract semilinear equations with application to semilinear wave and plate control problems [L-T.11].

Plate equations

In a recent paper [L-T.7], first announced in [L-T.6], the principal investigators have solved an open problem, raised by J. L. Lions, on exact controllability of a plate equation with control action only in the Dirichlet boundary conditions, while the corresponding Neumann boundary conditions are homogeneous. Further work by the principal investigators on plates with different boundary conditions is included in [L-T.9] (maximal, optimal regularity) and in [L-T.8] (exact controllability). Further results on the case where the control acts only in the bending moment were recently obtained by the principal investigators in [L-T.10], [L.5] for Euler-Bernoulli models and in [L-T.12] for Kirchoff models.

(ii) Boundary feedback stabilization

Linear case: uniform stabilization

The principal investigators results on uniform stabilization for the wave equation with feedback in the Dirichlet boundary conditions was already noted above [L-T.1]. In addition we cite a new proof which provides further insight into J. Lagnese's result on uniform stabilization with feedback in the Neumann boundary conditions given by the principal investigators in [T.1]. Uniform stabilization results were recently obtained for plates under a variety of boundary conditions including the case where only one feedback control is used out of the two available ones: see [B-T.1] (which is based on J. Bartolomeo's Ph.D. dissertation[1] and [O-T.1] (which contains material of N. Ourada's Ph.D. dissertation[1] both written under R.T.) and [L-T.10], [L-T.12], the latter references covering the case where the feedback control acts only in the bending moment in the Euler-Bernoulli and in the Kirchoff model respectively. A sharp result on the lack of uniform stabilization was also achieved in [T.3]. Finally, a substantial improvement of the uniform stabilization result for wave equations with Dirichlet control in [L-T.1] has been recently obtained in [L-T.13], in the sense that geometrical conditions on the spatial domain have been completely removed.

Nonlinear case: strong stabilization

Strong stabilization results by the principal investigators for waves and plates with nonlinear boundary damping defined by monotone functions are given in [L.1-L.4], [L.7 - L.9]. These results extend, by means of a general technique applicable to spatial domains of any dimension, recent results of the literature obtained only in the one-dimensional case.

---

[1]J. Bartolomeo received his Ph.D. degree in mathematics from the University of Florida, in December 1988.

[2]N. Ourada is expected to receive his Ph.D. in Applied Mathematics from the University of Virginia by September 1990.

Nonlinear case: uniform exponential stabilization

The following question is addressed and solved in [L.2]: does a linear stabilizing feedback operator provide a robust mechanism (at least locally) in the presence of nonlinear undesirable perturbations? An affirmative answer is given and applied to waves and plates in feedback form. Other nonlinear problems are solved in [L.4 - L.7].

(iii) Optimal quadratic cost problem over an infinite horizon

The principal investigators have recently completed, in collaboration with F. Flandoli, a rather thorough and comprehensive study on the optimal quadratic cost problem over an infinite horizon for an abstract unifying model which includes waves and plates [L-T.5]. This work extends prior work by the principal investigators on the same problem for wave equations with Dirichlet boundary control (SIAM J. Control, 1985).

(iv) Structural damping for elastic systems

As already noted in the proposal (continuation) submitted to the AFOSR in January 1987, work by the principal investigators in collaboration with S. Chen, has succeeded in proving two conjectures, raised by G. Chen and D. Russell in 1982, that if the damping operator behaves like $A^{1/2}$ (in a technical sense), where A is the elastic operator, then the elastic system displays structural damping, see [C-T.1]. Further work by the same authors shows that the same conclusion holds if the damping operator behaves like A^{α}, $1/2 \leq \alpha \leq 1$ see [C-T.2]. Applications of these results to physically significant models of plates are also given. Indeed, even a more general result holds true. The result is false if, instead, $0 \leq \alpha < 1/2$. However, in this range $0 < \alpha$ $1/2$, the semigroup is differentiable for all positive times [C-T.3], in fact even of Gevrey class $\delta > 1/2\alpha$. Regularity of structurally damped plates with point control and boundary control are given

in [T.6].

(v) Numerical aspects

Work in the area of numerical analysis can be divided into two parts, reviewed separately below.

Numerical approximations of Riccati equations.

The ultimate goal of proving exponential stability, which is uniform with respect to the parameter of approximation, of stabilizing feedbacks based on the algebraic Riccati equations was achieved by the principal investigators in the case (i) of arbitrary analytic semigroups with (strongly) unbounded feedbacks, see [L-T.14], (ii) in the general case of semigroups [L-11]. Examples include: parabolic equations with Dirichlet boundary controls already treated in [L-T.15], strongly damped wave equations with boundary control, wave equations with boundary control, and Kirchhoff plate equations with boundary/point control.

Numerical schemes for the computation of the solutions of wave equations with rough (nonsmooth) date

Convergence and stability of the algorithm, along with numerical illustrations and examples are obtained in [L-S.1-3]. Numerical analysis for delay differential equations are given in [L-M.1] and for parabolic equations in [Ch-L].

(vi) Robust stabilization

The affect of uncontrolled (generally nonlinear) perturbations on the stability properties of the closed loop system was studied in [L-10], [L-9], see also [L-S.4]. It was shown in [L.10] that the stabilization via Riccati feedback provides particularly desirable robustness properties.

SECTION II

SUMMARY OF SCIENTIFIC PROGRESS

List of Recent Work by the Co-Principal Investigators (I. Lasiecka and R. Triggiani) related to AFOSR Grant 87-0321

A. Work by I. Lasiecka and G. Choudoury:

[Ch-L.1] Semidiscrete Approximation of Parabolic Boundary Value Problems with Nonsmooth Boundary Data. Submitted *SIAM J. Numerical Analysis*. Presented at 1988 SIAM Annual Conference in San Diego.

B. Work by I. Lasiecka and J. Sokolowski:

[L-S.1] Finite element approximations of wave equation with Dirichlet boundary data defined on a bounded domain in R^2 (jointly with P. Neitteanmaki). Lecture notes in Control and Information Sciences, 102. Distributed Parameter Systems, ed. by F. Kappel et al., Springer-Verlag, 1987, pp. 216-234.

[L-S.2] Regularity and strong convergence of variational approximations to nonhomogeneous Dirichlet hyperbolic boundary value problems, *SIAM Journal of Mathematical Analysis*. Vol. 19, No. 3, 1988.

[L-S.3] Semidiscrete approximations of hyperbolic boundary value problems with nonhomogeneous Dirichlet boundary conditions, *SIAM Journal of Mathematical Analysis*, Vol. 20, No. 6, pp. 1366-1387 (1989).

[L-S.4] Sensitivity Analysis of Optimal Control Problems for Wave Equations *SIAM J. Control,* short version CDC Proceedings 1989.

C. Work by I. Lasiecka and A. Manitus

[L-M] Differentiability and convergence rates at approximating semigroups for related functional differential equations. *SIAM J. Numer. Anal.* Vol. 25, No. 4, 1988.

D. Work by I. Lasiecka

[L.1] Strong stabilization of a nonlinear wave equation with dissipation on the boundary and related problems, *Recent Advances in Communication and Theory,* ed. by R. E. Kalman, et al., Optimization Software Inc., Publication Division, New York, 1987, pp. 194-213.

[L.2] Boundary stabilization of hyperbolic and parabolic equations with nonlinearly perturbed boundary conditions, *J. Differential Equations.* Vol. 75, No. 1, 1988.

[L.3] Exponential local stability of first order strictly hyperbolic systems with nonlinear perturbations on the boundary. *Lecture Notes in Control Sciences,* Springer-Verlag, ed. J. Zolesio 100, pp. 212-235, 1988.

[L.4] Stability of wave equations with nonlinear damping in the Dirichlet and Neumann boundary conditions. Lecture Notes in *Control and Information Sciences,* Springer-Verlag, Vol. 125, pp. 47-65 (1989).

[L.5] Exact controllability of a plate equation with one control acting as a bending moment. To appear, Marcel Dekker.

[L.6] Controllability of viscoelastic plates *International Series of Numerical Mathematics*, Vol. 91, Birkhauser 1988, pp. 237-248.

[L.7] Asymptotic behavior of the solutions of the Kirkhoff plate with nonlinear dissipation in the bending moment, presented at the IFIP Conference at Clermont-Ferrand. Lecture Notes in *Control and Information Sciences*, Vol. 125, Springer-Verlag 1988, pp. 168-176.

[L-8] Asymptotic behavior of solutions to plate equations with nonlinear dissipation occurring through shear forces and bending moments. *Applied Mathematics and Optimization*, Vol. 21, pp. 167-191 (1990).

[L-9] Stabilization of the semilinear wave equation with viscous damping, to appear *Journal Diff. Eq.*

[L.10] Exponential stabilization of hyperbolic systems with nonlinear, unbounded perturbations - · Riccati operator approach, submitted to *Applicable Analysis*.

[L.11] Approximations of solutions to infinite dimensional Algebraic Riccati Equations with unbounded input operators, submitted to *Numerical Functional Analysis*.

E. Work by I. Lasiecka and R. Triggiani

[L-T.1] Exponential uniform energy decay rates of the wave equation in a bounded region with $L_2(0, \infty; L_2(\Gamma))$ - boundary feedback in the Dirichlet Boundary Conditions, *J. Diff. Eqts.*, 66 (1987), 340-390.

[L-T.2] Trace regularity of the solutions of the wave equation with homogeneous Neumann

boundary conditions and data supported away from the boundary, *J. Mathem. Anal. and Applications,* Vol. 141, No. 1, (1989), pp. 49-71.

[L-T.3] Exact boundary controllability for the wave equation with Neumann boundary control, *Applied Mathem. and Optimization,* Vol. 19, (1989), pp. 243-290.

[L-T.4] A lifting theorem for the time regularity of solutions to abstract equations with unbounded operators and applications to hyperbolic equations, *Proc. Amer. Math. Soc.,* Vol. 104, No. 3, (1988), pp. 745-755.

[L-T.5] Algebraic Riccati equations with non-smoothing observation arising in hyperbolic and Euler-Bernoulli boundary control problems (jointly with F. Flandoli), *Annali Mathem. Pura e Appl.,* Vol. CLIII, (1988), pp. 307-382.

[L-T.6] Exact controllability of the Euler-Bernoulli equation with L_2(SUM) - control only in the Dirichlet boundary condition, *Rendiconti Accademia Nazionale dei Lincei Classe di Scienze Fisiche e matematiche.* Vol. LXXXI, 1987.

[L-T.7] Exact controllability of the Euler-Bernoulli Equation with Controls in the Dirichlet and Neumann boundary conditions: a non-conservative case, *SIAM J. Control,* Vol. 27, No. 2, (1989), pp. 330-373.

[L-T.8] Exact controllability of the Euler-Bernoulli equation with boundary controls for displacement and moments, *J. Math. Anal. and Applications,* to appear.

[L-T.9] Infinite horizon quadratic cost problems for boundary control problems, *Proceedings CDC Conference,* Los Angeles, Calif., December 1987.

[L-T.10] Exact controllability and uniform energy decay rates for Euler-Bernoulli equations with control only in the bending moment, presented at Workshops in Vorau (Austria) July 1988; Pisa (Italy) July 1988, at Amer. Math. Soc. Meeting, University of Kansas, October 1988; at Southeast Conference in Differ. Equations, University of Georgia, Athens, November 1988; at CDC Conference, Austin, Texas, December 1988.

[L-T.11] Exact controllability of semilinear abstract systems with applications to waves and plates boundary control problems *Appl. Math. & Optimiz.*, to appear. Presented at CDC Conference, Tampa, December 1989.

[L-T.12] Exact controllability and uniform stabilization of Kirchoff plates with boundary control only on $\Delta w|_{\Sigma}$; preprint 1989, presented at CDC Conference, Tampa, December 19898.

[L-T.13] Uniform stabilization of the wave equation without geometrical conditions, to appear in Springer-Verlag Lectures Notes 1990; Presented at CDC Conference, Tampa, December 1989.

[L-T.14] Approximations of infinite horizon quadratic cost problems for abstract systems modelled by analytic semigroup, preprint 1990.

F. <u>Work by Shuping Chen and R. Triggiani</u>

[C-T.1] Proof of two conjectures of G. Chen and D. L. Russell on structural damping for elastic systems: The case $\alpha = ½$. Springer-Verlag Lecture Notes in Mathematics, #1354, pp. 234-256.

[C-T.2] Proof of extensions of two conjectures on structural damping for elastic systems; the case $\frac{1}{2} \leq \alpha \leq 1$, *Pacific J. of Mathematics*, Vol. 136, No. 1, (1989), 15-55.

[C-T.3] Differentiable and Gevrey class semigroups arising from elastic systems with gentle perturbation: the case $0 < \alpha < \frac{1}{2}$, to appear. Presented at CDC Conference Austin, Texas, December 1988. Proceedings American Mathem. Soc., to appear.

[C-T.4] Analyticity of elastic systems with non-selfadjoint dissipation: the general case, February 1989.

[C-T.5] Domains of fractional powers of certain elastic operators and applications, *J. Diff. Eqts*, to appear.

G. Work by Jerry Bartolomeo and R. Triggiani

[B-T.1] Uniform energy decay rates for Euler-Bernoulli equations with feedback operators in the Dirichlet/Neumann B. C., *SIAM J. Mathem. Anal.*, to appear. Presented at Workshops in Vorau (Austria) July 88; Pisa (Italy) July 88; at Amer. Math. Soc. Meeting (Specialized Session) University of Kansas, October 1988; at Southeast Conference in Differential Equations, University of Georgia, Athens, Georgia, November 1988; at CDC Conference Austin, Texas, December 1988.

H. Work by Norman Ourada and R. Triggiani

[O-T.1] Uniform stabilization of the Euler-Bernoulli equation with feedback operator only in the Neumann Boundary conditions, preprint 1989, submitted. Presented at Conference in Differential Equations, University of North Carolina, Charlotte,

October 1989.

I. Work by R. Triggiani

[T.1] Wave equation on a bounded domain with boundary dissipation: an operator approach, *J. Mathem. Anal. and Applications,* Vol. 137, (1989), 438-461.

Also, *Recent Advances in Communication and Control Theory,* volume in honor of A. V. Balakrishnan's 60th birthday, ed. by R. E. Kalman, G. I. Marchuck, etc. *Optimization Software,* New York, 1987, pp. 262-286.

Also, *Operator Methods for Optimal Control Problems,* ed. by S. Lee, Marcel Dekker, 1987, Vol. 108, pp. 283-310.

[T.2] Exact boundary controllability on $L_2(\Omega) \times H^{-1}(\Omega)$ of the wave equation with Dirichlet boundary control acting on a portion of the boundary and related problems, *Applied Mathem. and Optimiz.,* 18 (1988) 241-277. Also: Lecture Notes in Control Sciences, Springer-Verlag, No. 102, pp. 292-332, 1987.

[T.3] Finite rank, relatively bounded perturbations of semigroup generators. part III: A sharp result of the lack of uniform stabilization, *Internation J. of Different. and Integral Eqts.,* to appear. Springer-Verlag Lecture Notes in Control, Proceedings, INRIA Conference, Antibes (France) June 1988. Also: *Proceedings of First Conference on Communication and Control Theory,* Washington DC, June 1987, Optimization Software, pp. 123-146.

[T.4] Lack of uniform stabilization of non-contractive semigroups under compact

perturbation, Proc. Amer. Math. Soc., Vol. 105, (1989), 375-383.

[T.5] Exact controllability of wave and Euler-Bernoulli equations in the presence of damping, Proceedings International Conference on Differential Equations, Columbus, Ohio, March 21-25, 1988; and Proceedings of Conference 'Thirty years of modern optimal control theory," University of Rhode Island, Kingston, Rhode Island, June 1988; Marcel Dekker Lecture Notes in Pure and Applied Mathematics, Vol. 119, pp. 377-389.

[T.6] Regularity of structurally damped problems with point control and boundary control, *J. Math. Analy. & Applic.*, to appear.

[T.7] Uniform exponential energy decay of Euler-Bernoulli equations by suitable boundary feedback operators, *Intern. Series of Numerical Mathematics,* Vol. 91, 1989, Birkhauser, 391-400.

[T.8] Constructive exact controllability and finite rank conditions, to appear.

[T.9] A counter example to strong stability for dissipative operators, to appear.

DISTRIBUTION LIST

1 - 6*	Air Force Office of Scientific Research Building 410 Bolling Air Force Base Washington, DC 20332 Attention: Ms. Pat Bell
7 - 8	R. Triggiani, AM
9	I. Lasiecka, AM
10	J. G. Simmonds, AM
11 - 12	E. H. Pancake, Clark Hall
13	SEAS Preaward Administration Files

*One reproducible copy

JO#3030:ph